Avalanche consumption and the stationary regions of the density profile around the droplet in the theory of condensation

V.Kurasov

Victor.Kurasov@pobox.spbu.ru

The kinetic theory of the condensation was significantly developed during the last years. Creation of the theoretical description of the condensation in frames of the mean field approach started the based quantitative description of this process. The inclusion of the profiles of the vapor density around every droplet leads to the new view on the condensation. As it is shown in [1] one can apply the formalism of the Green function to see the form of the density profile around the droplet. Then the natural idea to see where the mean field approach can be applied can be considered.

The Green function of the diffusion equation has the known form

$$G \sim \exp(-\frac{r^2}{4Dt})/(Dt)^{3/2}$$

where r is the distance from the droplet, t is the characteristic time, D is the diffusion coefficient.

¿From the first point of view the characteristic distance of the stationary relaxed profile is given from the argument of the exponent in the expression for the Green function

$$r_{rel} \sim \sqrt{Dt}$$

The problem is to decide what time one has to put in the last formula. Naturally it is reasonable to put the characteristic duration of the period of the nucleation (or the formation of the main quantity of the supercritical embryos).

On the other hand the essential feature of the condensation process which allowed to formulate the property of the universality of the droplets size spectrum is the avalanche character of the vapor consumption by the droplet. This lies in contradiction with the last estimate.

The aim of this activity is resolve this problem and to give more specified estimate which allows to define concrete region of the stationarity of the vapor density profile around the droplet.

We shall start from the expression [1] for the difference of the supersaturation ζ from the ideal supersaturation Phi

$$\frac{\Phi - \zeta}{\Phi} = \sqrt{\frac{2}{\pi}} (\frac{v_l}{v_v})^{1/2} f(\beta)$$

where v_v and v_l are the volumes for the molecule in the vapor phase and in the liquid phase and $f(\beta)$ is the function of the argument

$$\beta = \frac{r}{\sqrt{4Dt}}$$

For the concrete form of $f(\beta)$ one can get

$$f(\beta) = \int_{\beta}^{\infty} (\frac{1}{\beta^2} - \frac{1}{x^2})^{1/2} \exp(-x^2) dx$$

In [2] it was shown that the mean field approach can be applied when parameter

$$\sigma = \Gamma^2 \frac{v_l}{v_v}$$

is small in comparison with unity[1] . In terms of σ one can rewrite the required expression as

$$\frac{\Phi - \zeta}{\Phi} = \sqrt{2\pi} \frac{\sigma^{1/2}}{\Gamma} f(\beta)$$

Obtain the exact expression for $f(\beta)$. One can rewrite the mentioned expression as

$$f(\beta) = 2\beta^2 \Gamma(3/2) exp(-\beta^2) \Psi(\frac{3}{2}, \frac{3}{2}; \beta^2)$$

Here $\Gamma(z)$ is the Gamma-function, $\Psi(x, y; z)$ is the confluent hypergeometric function.

The following asymptote for the great values of β can be gotten

$$f(\beta) = \exp(-\beta^2) \frac{1}{2\beta^3} \int_0^{\infty} y^{1/2} \exp(-y) dy \sim \frac{\exp(-\beta^2)}{\beta^3}$$

For small values of β one can get

$$f(\beta) = \frac{\sqrt{\pi}}{2} \frac{1}{\beta} \equiv f_{as}$$

This asymptote corresponds to the stationary solution for the density profile

$$\frac{\Phi - \zeta}{\Phi} \sim \sigma^{1/2} \frac{1}{\Gamma\beta}$$

[1] The parameter Γ is defined in [1].

The characteristic value of the error in the relative deviation of f from f_{as} have to attain the same value as the error of the calculation of the iterations, i.e. somewhere about 15 percents. Concrete calculations show such a deviation is already attained at $\beta \sim 0.05$. It means that the value of the parameter *sigma* have to be less than $1/400$. It means that the theory based on the stationary profile around the droplet can be applied only when $\sigma < 1/400$. So, the region of applicability becomes rather small and one has really use the formalism of the Green function described in [1].

It is the avalanche character of the droplets growth which leads to this great variation of the required value of the parameter σ necessary for the validity of the mean field approach. Note that the complete avalanche character would lead to the absence of the stationary region. We see that moderate avalanche character leads to the fact that the stationary (smaller than one can assume from the first point of view) region nevertheless exists. This leads to the applicability of the stationary approximation for the droplets growth. This fact is well known and based in many papers.

References

[1] Kurasov V.B. Physica A 226 (1996) 117-136

[2] Kurasov V.B. Preprint cond-mat@babbage.sissa.it Ref.: 9806342 (1997)